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AQA

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MM04 Mechanics 4

Mark Scheme

2008 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

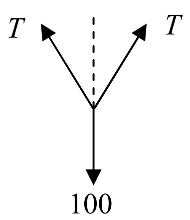
MM04

Q	Solution	Marks	Total	Comments
1(a)	Couple $\Rightarrow \sum$ horizontal component $=0$			
	$\sum$ vertical component $=0$			
	Vertically:			
	$2\sqrt{3}\cos 60° - Q\cos 30° = 0$	M1		$\sum$ vertical component $=0$
	$\therefore Q = 2$	A1		AG
	Horizontally:			
	$P - 2\sqrt{3}\sin 60° - Q\sin 30° = 0$	M1		$\sum$ horizontal component $=0$
		A1		one component correct (condone $\pm$)
	$\therefore P = 4$	A1	5	
(b)	Moments about B:			(N.B clockwise $-$ve/ anticlockwise +ve in solution below)
	$2\sqrt{3}\sin 60°(4) - 4(5)$	M1 A1√		Evidence of force $\times$ perp distance One term correct; ft error with P
	$=-8$			
	Magnitude $=8$	A1√	3	
	Or			
	Moments about A:			
	$-2\sqrt{3}\sin 60°(1) - 2\sin 30°(5)$	(M1A1)		Evidence of force $\times$ perp distance One term correct
	$=-8$			
	Magnitude = 8	(A1)		No ft for Q
	Or			
	Moments about C:			
	$-4(1) - 2\sin 30°(4)$	(M1A1√)		Evidence of force $\times$ perp distance One term correct, ft error with P
	$=-8$			
	Magnitude = 8	(A1√)		
	Or			
	Moments about centre of rod			
	$-P(2.5) - Q(2.5\sin 30°) + 2\sqrt{3}(1.5\sin 60°)$	M1A1√		Evidence of force $\times$ perp distance One term correct, ft error with P
	$=-8$			
	Magnitude = 8	A1√		
	Or			
	$\begin{bmatrix}0\\0\end{bmatrix}\times\begin{bmatrix}4\\0\end{bmatrix}+\begin{bmatrix}0\\-1\end{bmatrix}\times\begin{bmatrix}-3\\\sqrt{3}\end{bmatrix}+\begin{bmatrix}0\\-5\end{bmatrix}\times\begin{bmatrix}-1\\-\sqrt{3}\end{bmatrix}$	M1		Evidence of $\mathbf{r}\times\mathbf{F}$
	$=(0\ \ -3\ \ -5)\ \mathbf{k}$	A1√		one value correct
	$=-8\mathbf{k}$ Magnitude = 8	A1√		ft P value

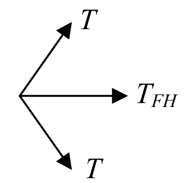
SC Max M1A0A0 for candidates who form an equation in part (b) without using a variable for couple

i.e. $4(2.5)+2\sqrt{3}(1.5\sin 60°)=2(2.5\sin 30°)$

MM04 (cont)

Q	Solution	Marks	Total	Comments
1(c)	Clockwise	B1√		ft answer (b) if directions all clear
2(a)	Magnitude $=100\,\text{N}$	B1		
	Whole system must be in equilibrium and force in *DE* must balance the $100\,\text{N}$ at *G*	E1	2	Reference to resolving whole system in equilibrium so $\sum F = 0$
(b)	Forces symmetrical about *FH* and *EG* $\Rightarrow$ equal magnitude	E(2,1,0)	2	E2 awarded for clear reference to **two axes** of symmetry
	Alternative As any joint in the framework is in equilibrium, so resultant force is zero At F resolve vert $T_{EF}\,sin\,60° = T_{FG}\,sin\,60°$ $\therefore T_{EF} = T_{FG}$ At H resolve vert $T_{EH}\,sin\,60° = T_{HG}\,sin\,60°$ $\therefore T_{HG} = T_{EH}$ At G resolve horiz $T_{GH}\,cos\,60° = T_{GF}\,cos\,60°$ $\therefore T_{GH} = T_{GF}$ Hence $T_{GH} = T_{EF} = T_{EH} = T_{FG}$	E(2,1,0)		
(c)	Consider forces at *G*, resolve vertically $T =$ Force in *FG* $=$ Force in *GH*  			
	$2T\cos 30° = 100$	M1		Attempt to resolve at *G* or *E* Correct equation formed
	$T \simeq 57.7\,\text{N}$	A1	2	$\frac{100}{\sqrt{3}}$ accepted

MM04

Q	Solution	Marks	Total	Comments
2(d)	Consider forces at H, resolve horizontally 			
	$T_{FH} + 2T\cos 60° = 0$	M1 A1√		Attempt to resolve at H or F Correct equation formed. Follow through error for T
	$\Rightarrow \lvert T_{FH} \rvert = 57.7\text{ N}$	A1√	3	Solved; condone ± Follow through error for T
(e)	*EH, EF, FG, HG* can be replaced by ropes They are all in tension **Or** *FH* can not be replaced by ropes It is the only one in thrust	B1 B1 B1 B1	 2	
	Total		**11**	
3(a)	$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$	B1	1	
(b)	$\overrightarrow{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & 2 & 2 \\ \mathbf{j} & 3 & -1 \\ \mathbf{k} & -6 & 4 \end{vmatrix}$	M1		Attempt at $\mathbf{r} \times \mathbf{F}$ or $\mathbf{F} \times \mathbf{r}$ M0 if no evidence of **i, j**, **k** components
	$= \begin{pmatrix} 6 \\ -20 \\ -8 \end{pmatrix}$	A2,1,0 √	3	One component correct = A1 Follow through $\overrightarrow{AB}$ [If $\mathbf{F} \times \mathbf{r}$ M1, A1, A0] max
(c)	$\sqrt{6^2 + 20^2 + 8^2} = \sqrt{500}$ $= 10\sqrt{5}\text{ N}$	M1 A1	 2	 AG must see $\sqrt{500}$ to award A1
(d)	$\sin\theta = \dfrac{10\sqrt{5}}{\left\lvert \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix} \right\rvert \left\lvert \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \right\rvert}$	M1		Use of $\sin\theta = \dfrac{\mathbf{a} \times \mathbf{b}}{\lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert}$ with correct vector pair
		B1		$\sqrt{49}$, 7 or $\sqrt{21}$ seen
	$= \dfrac{10\sqrt{5}}{7\sqrt{21}}$	A1√		Correct values ft their $\overrightarrow{AB}$
	$\theta \simeq 44°$	A1√	4	ft their $\overrightarrow{AB}$
	Total		**10**	

MM04

Q	Solution	Marks	Total	Comments
3(d)	SC if $90° - \theta$ found (wrong angle – correct triangle) ie $46°$ then award M1 B1 A1 A0 Max			
	Alternative			
3(d)	$\overrightarrow{AB}.\mathbf{F} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix} . \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = -23$	B1√		Their $\overrightarrow{AB}.\mathbf{F}$
	$\cos\theta = \dfrac{-23}{\left\lvert\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}\right\rvert \left\lvert\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}\right\rvert} = \dfrac{-23}{7\sqrt{21}}$	M1A1 √		use of $\cos\theta = \left\lvert\dfrac{a.b}{\lvert a\rvert\lvert b\rvert}\right\rvert$ with correct vector pair ft their $\overrightarrow{AB}$. (May not be explicitly seen)
	$\theta = \cos^{-1}\left(\dfrac{-23}{7\sqrt{21}}\right) = 135.8° \ldots$			
	$\therefore$ Required angle $= 180° - 135.8° = 44°$	A1√		ft their $\overrightarrow{AB}$

N.B Use of $\sin\theta / \cos\theta$ must be consistent with method chosen for M1

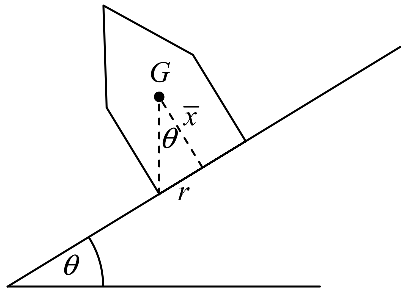
MM04 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$m = \pi r^2 \rho \Rightarrow \rho = \dfrac{m}{\pi r^2}$	B1		ρ and m linked – used anywhere
	Mass of elemental 'hoop' $= 2\pi\rho\,\delta x\, x$	M1		Attempt to consider elemental 'hoop' – mass correct
	MI of each hoop $= 2\pi\rho\,\delta x\, x^3$	A1		Use of mr^2 with elemental 'hoop'
	MI disc $= \int_0^r 2\pi\rho\,\delta x\, x^3 = \int_0^r \dfrac{2m}{r^2} x^3\,\mathrm{d}x$	m1		Attempt to integrate – dependant on first M1. Must be of form $\int kx^3 \mathrm{d}x$
	$= \left[\dfrac{2mx^4}{4r^2}\right]_0^r = \dfrac{mr^2}{2}$	A1	5	AG
(b)(i)	$\mathrm{MI}_{\text{disc}} = \dfrac{1}{2}mr^2 = \dfrac{1}{2}(200)(1.5)^2 = 225$	M1		Use of formula – either mr^2 or $\dfrac{1}{2}mr^2$
	$\mathrm{MI}_{\text{dom}} = mr^2 = 25(1.5)^2 = 56.25$	A1		Both correct
	Total $= 225 + 56.25 = 281.25$	A1	3	AG Evidence of $\mathrm{MI}_{\text{disc}} + \mathrm{MI}_{\text{dom}}$
(ii)	No (resultant) external forces	E1	1	
(iii)	Momentum conserved Momentum at start $= I\omega$ $= 281.25\left(\dfrac{\pi}{2}\right)$	M1		Attempt at angular momentum (either)
	Momentum at end $= 225\omega$	A1		Both correct
	$\Rightarrow 225\omega = 281.25\left(\dfrac{\pi}{2}\right)$	M1		Equation formed – cons. of momentum
	$\omega = \dfrac{5\pi}{8} = 1.96 \text{ rad s}^{-1}$	A1	4	CAO
	Total		**13**	

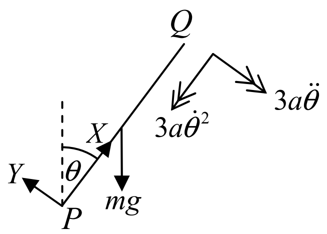
MM04 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\int_0^{2r} xy^2\,dx = \int_0^{2r} \frac{x^3}{4}\,dx$	M1		Attempt to use formula $\int xy^2 dx$
	$= \left[\frac{x^4}{16}\right]_0^{2r}$	A1		Integration correct
	$= r^4$			
	$\int_0^{2r} y^2\,dx = \int_0^{2r} \frac{x^2}{4}\,dx$			Or use of $\frac{1}{3}\pi r^2 h$ to get $\frac{2}{3}\pi r^3$
	$= \left[\frac{x^3}{12}\right]_0^{2r}$			
	$= \frac{2r^3}{3}$	B1		
	$\Rightarrow \bar{x} = r^4 \div \frac{2r^3}{3} = \frac{3r}{2}$	M1A1	5	AG use of $\bar{x} = \frac{\pi\int_0^{2r} xy^2 dx}{\pi\int_0^{2r} y^2 dx}$ NB – consistent use of π throughout for M1A1 at end (or cancelled at start)
(b)(i)	$\begin{array}{l\|r\|r} & \text{mass} & \text{distance} \\ \hline \text{Lower} & \pi r^2(2r)\rho & r \\ \text{Upper} & \frac{\pi r^2}{3}(2r)k\rho & 2r+\frac{r}{2} \end{array}$	B1		Any correct pairing seen anywhere (mass $\leftrightarrow$ distance)
	$\left(\pi 2r^3\rho + \frac{\pi 2r^3}{3}k\rho\right)\bar{x} = \pi 2r^3\rho(r)$	M1		Equation formed
	$+\frac{\pi 2r^3}{3}k\rho\left(\frac{5r}{2}\right)$	A2,1,0		lose 1 each 'type' of error
	$\Rightarrow \left(1+\frac{k}{3}\right)\bar{x} = r + \frac{5rk}{6}$			
	$\Rightarrow (6+2k)\bar{x} = (6+5k)r$			
	$\bar{x} = \left(\frac{6+5k}{6+2k}\right)r$	A1	5	Rearrange to obtain printed answer

MM04 (cont)

Q	Solution	Marks	Total	Comments
5(b)(ii)				
	$\tan\theta = \dfrac{r}{\bar{x}}$	M1 A1		Use of $\tan\theta$ Correct structure
	$\Rightarrow \dfrac{2}{3} = \dfrac{r}{\left(\dfrac{6+5k}{6+2k}\right)r}$	B1		Substitution of $\bar{x}, \tan\theta$
	$\dfrac{2}{3} = \dfrac{6+2k}{6+5k}$			
	$12 + 10k = 18 + 6k$	M1		Attempt to solve
	$4k = 6$			
	$k = \dfrac{3}{2}$	A1	5	
	Total		**15**	
6(a)(i)	$\dfrac{4}{3}m(3a)^2 = 12ma^2$	B1	1	
(ii)	Use conservation of energy			
	PE lost = KE gained			
	$mg3a(1-\cos\theta) = \dfrac{1}{2}(12ma^2)\dot{\theta}^2$	M1 A1,A1		Equation formed A1 each side
	$\dot{\theta}^2 = \dfrac{g}{2a}(1-\cos\theta)$	A1	4	AG
(iii)	Differentiate			
	$2\dot{\theta}\ddot{\theta} = \dfrac{g}{2a}(\sin\theta)\dot{\theta}$	M1		Attempt to differentiate – $\sin\theta$ seen $\Rightarrow$ M1
	$\ddot{\theta} = \dfrac{g}{4a}\sin\theta$	A1	2	$\dot{\theta}$ cancelled – clear indication
	Alternative			
6(a)(iii)	using $C = I\ddot{\theta}$ $\quad mg3a\sin\theta = 12ma^2\ddot{\theta}$	M1		
	$\therefore \ddot{\theta} = \dfrac{g\sin\theta}{4a}$	A1	2	

MM04 (cont)

Q	Solution	Marks	Total	Comments
6(b)(i)				
	Along PQ $mg\cos\theta - X = 3ma\dot{\theta}^2$	M1		Use of F = mass $\times$acc. along PQ M1 for either $(\pm mg\cos\theta \pm X)$ or $m(3a)\dot{\theta}^2$ or $\dfrac{m(3a\dot{\theta})^2}{3a}$
		A1		A1 fully correct
	$mg\cos\theta - X = 3ma\left[\dfrac{g}{2a}(1-\cos\theta)\right]$	A1		Use of (a)(ii) to replace $\dot{\theta}^2$
	$X = mg\cos\theta - \dfrac{3mg}{2} + \dfrac{3mg}{2}\cos\theta$ or $\dfrac{mg}{2}[5\cos\theta - 3]$	A1	4	Can be unsimplified
(ii)	Perpendicular to PQ $mg\sin\theta - Y = 3ma\ddot{\theta}$	M1		Use of $F =$ mass $\times$ acc perp to PQ, must have attempted both sides
	$mg\sin\theta - Y = 3ma\left[\dfrac{g}{4a}\sin\theta\right]$	A1√		Use of (a)(iii) to replace "their" $\ddot{\theta}$
	$Y = mg\sin\theta - \dfrac{3mg}{4}\sin\theta$ or $\dfrac{mg}{4}\sin\theta$	A1√	3	Follow through (a)(iii) (condone $\pm$ for b (i)(ii))
(c)	When Q is vertically below P $\theta = \pi$			
	$\Rightarrow Y = 0$	B1		Stated or implied
	$X = \dfrac{mg}{2}[-5-3] = -4mg$	M1		Substituting $\theta = \pi$
	$\Rightarrow$ magnitude of total force $= 4mg$	A1	3	CAO
(c)	**Alternative** Conservation of energy (at top) $\dfrac{1}{2}I\dot{\theta}^2 = mg6a$ $\therefore \dot{\theta}^2 = \dfrac{g}{a}$	B1		
	vertically $Y - mg = m3a\dot{\theta}^2$	M1		
	$Y - mg = 3mg$ $Y = 4mg$	A1		
	Total		**17**	
	TOTAL		**75**	